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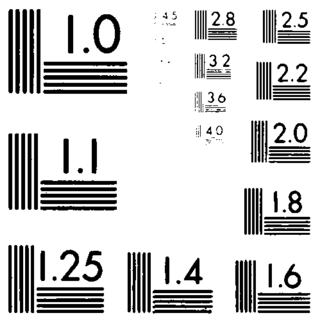

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Two Papers on the Methods of Evaluating the Quality of Solar Proton Event Forecasts From the People's Republic of China

ZHEN-XING CUI
SHAN-JIE QIAN

29 October 1979

Approved for public release; distribution unlimited.

Translated from Acta Astronomica Sinica (T'IEN-WEN HSUEH-PAO), Peking, China 16 (1): 6-11, 1975 and 17 (1): 115-117, 1976.

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FOR THE COMMANDER

John Howard

Chief Scientist

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Two recent scientific papers on evaluating the quality of solar proton event forecasting, published by scientists in the People's Republic of China, have been translated. The first paper, "A Method for Evaluating the Quality of Solar Proton Event Forecasts" by Zhen-xing Cui, reviews the common criteria used to evaluate the accuracy of solar proton event forecasts and discusses their defects. The concept of random forecast is introduced, and a combined indicator which can give a more reasonable assessment of the quality of

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20. Abstract (Continued)

forecasts is proposed. The second paper, "A Suggestion on the Improvement of the Criteria for Evaluating the Quality of Solar Proton Events Forecasts" by Shan-jie Qian, presents an improved indicator for evaluating the quality of solar proton event forecasts.

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Two Papers on the Methods of Evaluating the Quality of Solar Proton Event Forecasts From the People's Republic of China

PART 1

A Method for Evaluating the Quality of Solar Proton Event Forecasts

Zhen-xing Cui

1. INTRODUCTION

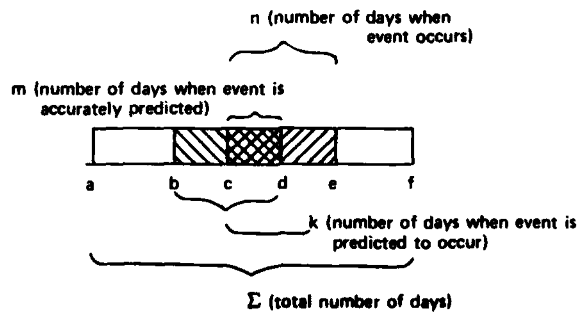
Solar forecasting involves the scientific prediction of solar proton events. To evaluate this kind of prediction, one has to select an appropriate indicator important for the design of forecast criteria, the improvement of forecasting methods, and the assessment of the work at hand.

The common indicators used have been the probability of accurate forecast (PAF) and the probability of erroneous forecast (PEF) of solar proton events and the PAF and the PEF of the safe (or quiet) period (that is, predicting the non-occurrence of solar proton events). However, it is recognized that there is a limit to the usefulness of these indicators. They cannot be used separately, nor can they be employed simultaneously. Thus, the question whether it is possible to have a combined indicator that gives an overall evaluation of the quality of forecasts always arises. This paper attempts to propose an answer — an equation that suggests a combined indicator. Surely, one can propose different combined indicators according to the nature of the problem and the requirements in practice. Nevertheless, a good combined indicator should be simple, and should have clear physical implications.

(Received for publication 25 October 1979)

2. PROBLEMS ASSOCIATED WITH THE COMMON INDICATORS

The result of the prediction of solar proton events can be represented by four independent basic quantities. They are Σ, the total number of days of observation; n, the number of days in Σ when a proton event occurs (or n_0, the number of occurrences); k, the number of days in Σ when a proton event is forecast to occur (or k_0, the number of occurrences that is predicted); and m, the number of days in Σ when a proton event is accurately forecast (or m_0, the number of occurrences that is accurately predicted). Since four independent basic quantities are involved, four indicators are generally required to evaluate the quality of the forecast. The four commonly used indicators are the PAF and the PEF of solar proton events and the PAF and the PEF of the safe period. The PAF of solar proton events is defined by the number of days when a proton event is accurately predicted divided by the number of days when a proton event occurs, while the PEF of solar proton events is defined by the number of days when a proton event is erroneously predicted divided by the number of days when a proton event is predicted to occur. Let Σ, n, k, and m be defined as above. Then, the number of days when a proton event is erroneously predicted is given by k - m. It should be noted here that the safe period or the nonoccurrence of events is the converse of the occurrence of events. It follows from the diagram below that the number of days in the safe period is given by $\overline{ac} + \overline{ef} = \Sigma - n$, the predicted number of days by $\overline{ab} + \overline{df} = \Sigma - k$, the number of accurately predicted days by $\xi = \overline{ab} + \overline{ef} = \Sigma - n - k + m$, and the number of erroneously predicted days by $\overline{de} = \Sigma - \xi - k$.



Let α and γ be the PAF and the PEF of solar proton events and λ and δ be the PAF and the PEF of the safe period. Then,

$$\alpha = \frac{m}{n} \tag{1}$$

$$\gamma = \frac{k - m}{k} \tag{2}$$

$$\lambda = \frac{\xi}{\Sigma - n} \tag{3}$$

$$\delta = \frac{\Sigma - k - \xi}{\Sigma - k} \quad . \tag{4}$$

The biases of these indicators are obvious. For example, an increase of k will result in a corresponding increase of α. In the extreme case, $\alpha = 1$ if proton event is predicted to occur every day, which is trivial. Similarly, in the case of the PEF, little or no prediction which is based solely on the accuracy of the indicator will result. If, however, all four indicators are used simultaneously, determination of the quality of the forecast is still difficult. Consider, for example, two forecasting methods, X and Y. The results obtained using method X are PAF = 60 percent and PEF = 40 percent and those with method Y are PAF = 40 percent and PEF = 20 percent. Although the PAF using method X is higher than that with method Y, the PEF is also higher, and therefore, it is difficult to conclude which one of these two methods is superior.

The usefulness of these indicators also depends on the frequency of proton events. If events are frequent within a certain time span, then the PAF will be high and the PEF will be low. Hence, the use of these indicators cannot reflect objectively the quality of the forecast. This point should be particularly noted in view of the fact that there will be a substantial increase in the number of proton events during the peak of solar activity. Furthermore, during such a high frequency of activity, the use of a single indicator does not have clear significance, since the chances of guessing the correct number of events can be very high. For example, if it is thought that almost 80 percent of every year falls within the safe period, it can be argued that a probability of 85 percent of accurate forecast is not necessarily a good indicator of quality prediction.

The above problems arise mainly because of the inherent limits of the individual indicators. Yet, the concurrent use of all four indicators lacks the criteria for mutual comparison.

3. RANDOM FORECAST

To overcome the difficulties encountered when the above-mentioned indicators are employed, we introduce here the concept of random forecast. This technique

is independent of the frequency of events and the predicted number of days, and the quality of the forecast work can be assessed objectively when the results are compared with those of the actual forecast. The results from different forecasts based on different criteria or algorithms can also be compared.

For convenience sake, we rename the probability of erroneous forecast (PEF) as the accuracy of the forecast, which is defined as accuracy = 1 - PEF. Let β and ϵ be the accuracy of forecast for proton event and safe period, respectively. Then

$$\beta = 1 - \gamma = \frac{m}{k} \tag{5}$$

$$\epsilon = 1 - \delta = \frac{\xi}{\Sigma - k} \tag{6}$$

From hereon, α, β, δ, and ϵ will be referred to as the sub-indicators.

Strictly speaking, random forecast has no physical interpretation. It simply makes use of the most probable outcome that would be obtained if an average probability is used.

Assume that the four basic quantities in forecast practice are Σ, n, k, and m, as defined previously, and the corresponding indicators are α, β, λ, and ϵ, as given in Eqs. (1), (5), (3) and (6), respectively. Then, for comparison, we will find the most probable result of k when a random forecast is used. Since a proton event occurs in n of Σ days of observation, it follows that the average probability of the daily occurrence of events is given by $P = n/\Sigma$. Let m' be the most probable number of days when a proton event is accurately predicted. Then, $m' = Pk = nk/\Sigma$. Thus, the four basic quantities for random forecast are Σ, n, k, and m', and the corresponding sub-indicators are α', β', λ', and ϵ'. It follows from previous definitions that

$$\alpha' = \frac{m'}{n} = \frac{k}{\Sigma} \tag{7}$$

$$\beta' = \frac{m'}{k} = \frac{n}{\Sigma} \tag{8}$$

$$\lambda' = \frac{\xi'}{\Sigma - n} = 1 - \frac{k}{\Sigma} \tag{9}$$

$$\epsilon' = \frac{\xi'}{\Sigma - k} = 1 - \frac{n}{\Sigma} \tag{10}$$

The ratio of the sub-indicators of actual forecast to the corresponding sub-indicators of random forecast gives a comparison on the quality of the forecast

$$A = \frac{\alpha}{\alpha'} = \frac{\alpha}{\eta} \tag{11}$$

$$B = \frac{\beta}{\beta'} = \frac{\alpha}{\eta} \tag{12}$$

$$C = \frac{\lambda}{\lambda'} = \frac{\lambda}{1 - \eta} \tag{13}$$

$$D = \frac{\epsilon}{\epsilon'} = \frac{\lambda}{1 - \eta} \tag{14}$$

where $\eta = k/\Sigma$ is the ratio of the number of days when a proton event is predicted to occur to the total number of days of observation. Obviously, the quality of the actual forecast is better than that of the random forecast only if $A = B > 1$ and $C = D > 1$. Otherwise, the forecast is meaningless since the best it can give is the same as that one would obtain by doing a random forecast, $A = B$ and $C = D$.

4. COMBINED INDICATOR

If we consider that proton-event and safe-period forecasts are of equal importance, then the product of A and C will give the ratio of the quality of the actual forecast to that of the random forecast. Hence, we can consider a combined indicator g such that

$$g = AC = \frac{\alpha\lambda}{\eta(1 - \eta)} . \tag{15}$$

The above discussion is based on the number of days. The corresponding equation based on the number of occurrences is given by

$$g_0 = \frac{\alpha_0 \lambda}{\eta(1 - \eta)} \tag{16}$$

where $\alpha_0 = m_0/n_0$, with m_0 being the number of occurrences of events that is accurately predicted and n_0 being the number of occurrences. The symbols λ and η are the same as those defined previously.

Let us now consider the following properties of g:

(1) The meaning of g is explicit. From the above derivations, we can see that g is the overall ratio of the quality of an actual forecast to that of a random forecast. The actual forecast is useful only if $g > 1$. The larger the value of g implies, the more accurate the forecast.

(2) The combined indicator g is applicable even in the extreme cases. For example, while the PAF is equal to one if a proton event is predicted to occur everyday, the PAF of the safe period is zero, which is meaningless. However, the value of g in this case is indeterminate and can be treated as a singular point ($g = 0/0$), and therefore, g can eliminate the trivial results implied by the sub-indicators under extreme conditions.

(3) The combined indicator g can be used not only to assess the quality of the forecast but also to determine whether events and the criteria used are positively, negatively or zero correlated. To further this point, we first prove a property of g, namely, the necessary and sufficient condition for $g > 1$ is $\alpha > \eta$. The proof is as follows:

If $\alpha > \eta$, that is, $m/n > k/\Sigma$ and $\Sigma m > nk$, then

$$\Sigma k - nk > \Sigma k - \Sigma m, \frac{k}{\Sigma} > \frac{k - m}{\Sigma - n}$$

and

$$1 - \frac{k}{\Sigma} < 1 - \frac{k - m}{\Sigma - n}$$

that is

$$1 - \eta < \lambda, \frac{\lambda}{1 - \eta} > 1 \quad .$$

Consequently

$$g = \frac{\alpha\lambda}{\eta(1 - \eta)} > 1 \quad .$$

Similar reasonings will show that $g < 1$ if $\alpha < \eta$.

The above property indicates that the forecast is meaningful only if the ratio of the number of days when a proton event is predicted to occur to the total number of days of observation is smaller than the accuracy of the forecast. This is commonly known as the degree of success in forecast practice. For example, if a forecast is done using 30 percent of the total number of days, then the degree of

the forecast is high if the accuracy is 40 percent; the degree of success is low if the accuracy is 20 percent.

In which follows, we shall discuss the ways of determining the properties of the forecast criteria. Let S denote solar proton events and R the forecast criteria. Then, using the same Σ, n, k, and m notations and provided that R holds true, k is the number of days when a proton event is forecast to occur and m is the number of days when S is accurately predicted. Therefore, the probability of occurrence of S based on R is $P(S|R)$ where

$$P(S|R) = \frac{m}{k} \ . \tag{17}$$

On the other hand, it is known from the theory of probability that the conditional probability of S with respect to R is $P_0(S|R)$, given by

$$P_0(S|R) = \frac{P(S \cdot R)}{P(R)} \tag{18}$$

where P(R) is the probability of occurrence of R and $P(S \cdot R)$ is the probability of simultaneous occurrence of S and R. If R and S are zero correlated, then

$$P(S \cdot R) = P(S)P(R) \tag{19}$$

where P(S) is the probability of occurrence of S. Obviously,

$$P(S) = \frac{n}{\Sigma} \ . \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (18), we get

$$P_0(S|R) = \frac{n}{\Sigma} \ . \tag{21}$$

If $P(S|R) = P_0(S|R)$, then R and S are zero correlated. R and S are positively correlated if $(P(S|R) > P_0(S|R)$ and negatively correlated if $P(S|R) < P_0(S|R)$. These values, according to Eqs. (18) and (21), correspond, respectively, to $\alpha = \eta$, $\alpha > \eta$, and $\alpha < \eta$. Hence, according to the above-mentioned properties of g, R is zero when $g = 1$; R is positive when $g > 1$ and negative when $g < 1$.

(4) Under the conditions given to R mentioned above, g and α/η are equivalent. Let us now examine g and α/η under different conditions, since, from these differences, it can be shown that the use of g can provide a more reasonable assessment of the quality of forecasts.

For example, method X makes use of 30 percent of the total number of days of observation (η_X = 30 percent) and the accuracy of forecast is 60 percent (α_X = 60 percent), while method Y employs 40 percent of the total number of days of observation (η_Y = 40 percent) and the accuracy is 80 percent (α_Y = 80 percent). In terms of the ratio α/η, the results are equal; however, they are not equal if g is used, that is,

$$\frac{g_Y}{g_X} = \frac{\Sigma - \frac{n}{3}}{\Sigma - \frac{4n}{7}} > 1 \quad .$$

This shows that the quality of the forecast using method Y is better than that using method X, which is reasonable, since the accuracy is 20 percent higher as compared to the 10 percent more time used in the forecast. The use of α/η does not show which one of these two methods is superior.

(5) We will now discuss the problem of normalization of g. Since generally the value of g (or g_0) is greater than 1, this does not conform with the usual practice of using a percentage to represent the quality of forecasts. Moreover, what is usually concerned is how far the forecast is from the best possible. Hence, there is the necessity of normalizing g.

The best possible forecast is when k n m, that is, when both the number of days when a proton event occurs and the number of days of the safe period are accurately predicted. Then, the combined indicator g takes on the maximum value

$$g_{mu} = \frac{\Sigma^2}{n(\Sigma - n)} \quad . \tag{22}$$

With a random forecast, g = 1 and we can let G be the normalized combined indicator, given by

$$G = \frac{g - 1}{g_{mu} - 1} \quad . \tag{23}$$

The corresponding equation based on the number of occurrences is given by

$$G_0 = \frac{g_0 - 1}{g_{mu} - 1} \quad . \tag{24}$$

Obviously, the quality of an actual forecast is better than that of a random forecast only if $G > 0$. The maximum value $G = 1$ represents the best forecast quality. $G < 0$ implies that the criteria is negative, and the forecast approach should be reversed.

(6) The above discussion has been based on the assumption that proton-event and safe-period forecasts are equally important. If, for the sake of avoiding "misses," emphasis on proton-event prediction is desired, then the combined indicator before normalization can be written as

$$g = A^x C^y = \left(\frac{\alpha}{\eta}\right)^x \left(\frac{\lambda}{1 - \eta}\right)^y \tag{25}$$

where both x and y are greater than or equal to 1. These weight indices (x and y) depend on the practical requirements of proton-event and safe-period forecasts. The maximum value of g then is given by

$$g_{mu} = \frac{\Sigma^{x+y}}{n^x(\Sigma - n)^y} \tag{26}$$

and the normalized combined indicator by

$$G = \frac{g - 1}{g_{mu} - 1} \quad . \tag{27}$$

It is hoped that the use of the combined indicator G will lead to a significant improvement in forecast quality.

PART 2

A Suggestion on the Improvement of the Criteria for Evaluating the Quality of Solar Proton Event Forecasts

Shan-jie Qian

In the article "A method for evaluating the quality of solar proton event forecasts" by Cui Zhen-xing,[1] the concept of random forecast was introduced and a combined indicator for assessing the quality of forecasts was suggested. The present author found that this combined indicator has certain defects in its application, and some suggestions for improvement will be presented in this article.

Four basic quantities are involved in predicting solar proton events. They are Σ, n, k, and m (whose meaning is the same as in Cui's article).* In assessing the quality of different forecasting methods, Σ and n are common and fixed so only two independent quantities, k and m, are required to be considered. These two indicators (for example, α and γ) are usually used to illustrate the quality of the forecasts. In these cases, λ and δ will not be independent indicators and can be expressed in terms of α and γ. The two indicators used in Cui's article were in fact (A, C) instead of (α, γ). In practice, a single combined indicator which enables a comparison of different methods to be made in a simple and reasonable manner is preferred to using two separate indicators. For this reason, in Cui's

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1. Chu, Zhen-xing (Tsu, Chen-hsing) (1975) A method for evaluating the quality of solar proton event forecasts, Acta Astron. Sinica 16(1):6-11. English translation: ID(RS)I-1198-76, Foreign Technology Division (AFSC), U.S. Air Force.

*The meaning of the symbols appearing in the article is given in the Appendix by the translator.

article, A and C were combined to form a single indicator g = AC. However, this indicator has the following three defects:

(1) Because solar proton events are small probability events, C usually approaches 1 and g almost equals A;

(2) the indicator A only reflects the average of success in forecasting the occurrence of solar proton events and is unable to provide any indication of the frequency of "misses." Thus, in Cui's article the numbers of occurrences of solar proton events missed by methods X and Y are different but their values of A are equal. When the combined indicator g is used the probability of the events missed is reflected by C which, however, is not very sensitive;

(3) because of (2), we cannot, by raising the power of A (for example, by defining $g = A^x C$, $x > 1$), minimize the number of events missed by the forecasts.

The above defects of g can be clearly seen from Table 1. We have chosen (A, α) as independent quantities with the corresponding values of g listed (taking $\Sigma = 100$ and $n = 10$). It can be seen from this table that g is slightly greater than A and is not sensitive to variations in α. As a result, the isolines g = constant (that is, isopleths of equal forecasting skill or quality) are close to the A = constant lines. It is not difficult to see from this table the validity of the statement given in (3) above.

Table 1. Values of g = AC (with $\Sigma = 100$, $n = 10$)

A \ α	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0											
1		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2		2.01	2.02	2.04	2.06	2.07	2.10	2.12	2.15	2.18	2.22
3		3.02	3.05	3.08	3.10	3.13	3.17	3.20	3.24	3.29	3.33
4		4.04	4.07	4.11	4.15	4.19	4.24	4.28	4.33	4.39	4.44
5		5.05	5.10	5.14	5.20	5.25	5.30	5.36	5.43	5.49	5.56
6		6.05	6.11	6.17	6.24	6.31	6.37	6.44	6.51	6.59	6.67
7		7.07	7.14	7.21	7.28	7.36	7.43	7.52	7.60	7.69	7.78
8		8.08	8.16	8.24	8.33	8.42	8.50	8.60	8.69	8.79	8.89
9		9.09	9.18	9.28	9.37	9.47	9.58	9.68	9.78	9.89	10.0
10		10.1	10.2	10.3	10.4	10.5	10.6	10.8	10.9	11.0	11.1

To overcome these defects the following indicators are proposed and considered, which appear to provide a more reasonable assessment.

In assessing the quality of solar proton event forecasts let us use the indicator

$$D_1 = A\alpha$$

(here, we assume $A > 1$, that is, we do not consider those methods which are worse then random forecasts with $A \leq 1$).

In assessing the quality of forecasts of tranquil or quiet periods (that is, predicting the non-occurrence of solar proton events) let us use the indicator

$$D_2 = C\lambda$$

We shall define our combined indicator as

$$D = D_1 D_2 = AC\lambda\alpha = g\lambda\alpha = \left[\frac{M}{N}\left(1 - \frac{K - M}{1 - N}\right)\right]^2 \frac{1}{K(1 - K)}$$

(where $N = n/\Sigma$, $M = m/\Sigma$, and $K = k/\Sigma$). The values of D are listed in Table 2. The isopleths D = constant are from a family of hyperbolas. They reflect the degrees of success as well as chances of misses of a method in forecasting the occurrences of solar proton events. In the two extreme cases of "forecasting the occurrence of solar proton events every day" and "forecasting absence of activity every day," D does not take on an odd value but assumes a value of zero.

Of course, we may form other combined indicators to emphasize some particular aspect relating to the quality of the forecasts by the power raising technique. However, these indicators have to be functions of (M, K) and the way to combine two independent indicators to form a single indicator is dictated by the generally accepted view as to what form the isopleths of forecast skill or quality should take.

Table 2. Values of $D = AC\lambda_\alpha$ (with $\Sigma = 100$, $n = 10$)

A \ α	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	0*										
1		(0.09)	(0.16)	(0.21)	(0.24)	(0.25)	(0.24)	(0.21)	(0.16)	(0.09)	0†
2		0.19	0.37	0.53	0.68	0.81	0.92	1.02	1.11	1.18	1.23
3		0.29	0.58	0.76	1.11	1.36	1.60	1.84	2.06	2.27	2.47
4		0.40	0.79	1.17	1.55	1.92	2.29	2.65	3.01	3.36	3.70
5		0.50	1.00	1.49	1.99	2.48	3.00	3.46	3.95	4.45	4.94
6		0.60	1.20	1.81	2.42	3.04	3.65	4.28	4.90	5.54	6.17
7		0.71	1.41	2.13	2.86	3.59	4.33	5.09	5.85	6.62	7.41
8		0.81	1.62	2.45	3.29	4.15	5.02	5.90	6.80	7.71	8.64
9		0.91	1.83	2.77	3.73	4.71	5.70	6.72	7.75	8.81	9.87
10		1.01	2.04	3.09	4.17	5.27	6.38	7.53	8.70	9.89	11.1‡

*Including forecasts of non-occurrence of solar proton events (absence of activity) every day.

†Occurrence of solar proton event forecast every day.

‡Best forecasts.

Appendix

Definitions

The following is extracted from the article "A method for evaluating the quality of solar proton event forecasts" by Cui Zhen-xing [Acta Astronomica Sinica, 16(1):6-11, 1975]:

Σ is the total number of days of observation,

n is the number of days (in Σ) when solar proton event occurred,

k is the number of days (in Σ) when solar proton event was forecast to occur,

m is the number of days (in Σ) when occurrence of solar proton event was correctly forecast,

$\alpha = m/n$ is the fraction of days in n, when occurrence of solar proton event was correctly forecast,

$\gamma = k - m/k$ is the fraction of days in k, when occurrence of solar proton event was not forecast,

$\xi = \Sigma - n - k + m$ is the number of days (in Σ) when non-occurrence of solar proton event (that is, safe) was correctly forecast,

$$\lambda = \frac{\xi}{\Sigma - n} \ ,$$

$$\delta = \frac{\Sigma - k - \xi}{\Sigma - k} \ ,$$

$\eta = k/\Sigma$ is the fraction of days in Σ when solar proton event was forecast to occur,

$$A = \frac{\alpha}{\eta} \ ,$$

$$C = \frac{\lambda}{1 - \eta} \ .$$